

$$\int x \log x \, dx = \log x \int x \, dx - \int \left(\frac{d(\log x)}{dx} \cdot \int x \, dx \right) dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \log x \cdot \frac{x^2}{2} - \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$= \frac{x^2}{4} (2 \log x - 1) + c$$