Operations with Complex Numbers

Complex numbers are numbers that consist of a real part and an imaginary part. The imaginary part is an , (which is $\sqrt{-1}$), with a real number coefficient in front of it. Complex numbers are usually written in the form a + b . When you work with complex numbers, you can treat the as if it were a variable.

OPERATIONS

To add or subtract complex numbers, add the real parts, and add the coefficients on .

Example 1: Add $(1 + 2i) + (3 + 4i)$.
Solution:

$$\begin{aligned}(1 + 2i) + (3 + 4i) &= (1 + 3) + (2 + 4)i \\ &= 4 + 6i\end{aligned}$$

To multiply complex numbers, foil out the numbers, and then convert ^{2} into −1.

Example 2: Multiply $(1 + 2i) \cdot (3 + 4i)$.
Solution:

$$\begin{aligned}(1 + 2i) \cdot (3 + 4i) &= 1 \cdot 3 + 1 \cdot 4i + 2i \cdot 3 + 2i \cdot 4i \\ &= 3 + 4i + 6i + 8i^2 \\ &= 3 + 10i + 8 \cdot (-1) \\ &= -5 + 10i\end{aligned}$$

To divide complex numbers, write the division problem as a fraction and then "rationalize" the denominator by multiplying by the conjugate. (The conjugate of a + b is a − b and vice versa.) Remember to express your final answer in the correct form.

Example 3: Divide $(3 + 4i) \div (1 + 2i)$.
Solution:

$$\begin{aligned}\frac{3+4i}{1+2i} &= \frac{3+4i}{1+2i} \cdot \frac{1-2i}{1-2i} \\ &= \frac{3+4i-6i-8i^2}{1-2i+2i-4i^2} \\ &= \frac{3-2i-8(-1)}{1-4(-1)} \\ &= \frac{11-2i}{5} \\ &= \tfrac{11}{5} - \tfrac{2}{5}i\end{aligned}$$

Authored by Gordon Wong