3 Linear viscoelasticity

A **linear viscoelastic fluid** is a fluid which has a linear relationship between its strain history and its current value of stress:

$$\sigma(t) = \int_{-\infty}^{t} G(t-t')\dot{\gamma}(t')\ \mathrm{d}t'$$

The function $G(t)$ is the relaxation modulus of the fluid. Because a fluid can never remember times in the future, $G(t) = 0$ if $t < 0$.

Physically, you would also expect that more recent strains would be more important than those from longer ago, so in $t > 0$, $G(t)$ should be a decreasing function. There aren't really any other constraints on G.

A few often-used forms for $G(t)$ where $t \geq 0$ are:

Single exponential	$G_0 \exp[-t/\tau]$
Multi-mode exponential	$G_1 \exp[-t/\tau_1] + G_2 \exp[-t/\tau_2] + \cdots$
Viscous fluid	$\eta\,\delta(t)$
Linearly elastic solid	G_0

Let's just check the last two. For the viscous form we have

$$\sigma(t) = \int_{-\infty}^{t} G(t-t')\dot{\gamma}(t')\ \mathrm{d}t' = \int_{-\infty}^{t} \eta\,\delta(t-t')\dot{\gamma}(t')\ \mathrm{d}t' = \eta\dot{\gamma}(t)$$

as we would expect for a Newtonian viscous fluid. For the elastic solid we have

$$\sigma(t) = \int_{-\infty}^{t} G(t-t')\dot{\gamma}(t')\ \mathrm{d}t' = \int_{-\infty}^{t} G_0\dot{\gamma}(t')\ \mathrm{d}t' = G_0\int_{-\infty}^{t} \dot{\gamma}(t')\ \mathrm{d}t'.$$

The integral on the right is the total strain (or shear) the material has undergone: so this, too, gives the form we expect.

3.1 Creep

Given a sample of material, how would you go about modelling it? Even if you start by assuming it is a linear material (and they all are for small enough strains), how would you calculate $G(t)$?

One way would be to carry out a **step strain** experiment: for shear flow this means you would set up your material between two plates and leave it to settle, so it loses the memory of the flow that put it there. Once it has had time to relax, you shear it through one shear unit (i.e. until the top plate has moved a distance the same as the distance between the plates). Then stop dead. Measure the force required to maintain the plates in position as time passes. The results will look something like this: