# Completing the Square

When asked to solve a quadratic equation, the strategy of isolating x doesn't work with both $x^2$- and x-terms. We have to factor. The sum-and-product method (or the ac method) is the fastest way, but doesn't work for all quadratics. For example,

$$x^2 + 3x - 17 = 0$$

It's obvious that there's no solution using the sum-and-product method: the only pairs of integers that multiply to give −17 are *1 and* −17 and −1 *and* 17, and neither pair adds up to 3. If this happens, we can solve the equation by completing the square.

*Example 1:* Solve: $5x^2 - x - 10 = 0$

*Solution:* Using the ac method, we need two numbers that multiply to give −50 and add up to −1. No two integers do that, so we can complete the square.

**[1] Move the constant to the other side of the equal sign.** We won't be using it, so let's get it out of the way.

$$5x^2 - x - 10 = 0$$
$$5x^2 - x = 10$$

**[2] If there's a leading coefficient on the $x^2$ term, factor it out.** This makes the problem simpler by reducing it to an $x^2$… question, rather than an $ax^2$… question. This may create fractions, but if the original polynomial doesn't factor neatly, there are going to be fractions in the answer.

$$5x^2 - x = 10$$
$$5(x^2 - \tfrac{1}{5}x) = 10$$

**[3] Determine what third term could go into the brackets to make a perfect trinomial square.** Take the coefficient in front of the x-term, divide it by two, and then square the result. If you're working with a fraction, the easiest way to divide by two is to double the denominator.

$$\tfrac{1}{5} \div 2 = \tfrac{1}{10} \text{ [*] remember this number!}$$
$$(\tfrac{1}{10})^2 = \tfrac{1}{100}$$

Pay attention to that asterisked number, $\tfrac{1}{10}$. We'll need it later….



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