# Section 3.1 – Polynomial Functions and Models

*Classifying Polynomials*:

- Recall: a **polynomial in one variable** is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$, where the $a_i$'s are real number **coefficients**. For nonzero $a_n$, the expression is said to be of $n$th degree (the highest power is $n$), the **leading term** is $a_n x^n$ and the **leading coefficient** is $a_n$
- Examples of polynomials that are common

| Degree | Name | Form |
|---|---|---|
| 0 | Constant | $f(x) = c$ |
| 1 | Linear | $f(x) = mx + b$ |
| 2 | Quadratic | $f(x) = ax^2 + bx + c$ |
| 3 | Cubic | $f(x) = ax^3 + bx^2 + cx + d$ |

- *Example. Classify* $f(x) = 15x^2 - 10 + 0.11x^4 - 7x^3$

  The highest power is 4, so it is a 4th degree polynomial. A quartic
  The leading term is $0.11x^4$

*Leading Behavior*:

- It is often useful to talk about a function as the independent variable (usually $x$) gets really large ( $x \to \infty$ ) or really small ( $x \to -\infty$ )
- As the independent variable gets really large in magnitude ( $x \to \pm\infty$ ), the leading term dominates the equation. All polynomials (except the constant function) will tend to $\pm\infty$ as $x$ gets large in magnitude. Why?
- To determine if it goes to $+\infty$ or $-\infty$, look at the behavior of the leading term, and evaluate it with a positive and a negative
- *Example, what happens to* $f(x) = -x^4 + 3x^2 + 3x$ *when* $x \to \pm\infty$ ?

  The leading term is $-x^4$
  As $x \to +\infty, \quad f \to -(+\infty)^4 = -\infty$
  As $x \to -\infty, \quad f \to -(-\infty)^4 = -\infty$
- *Example, what happens to* $f(x) = 12x^5 + 3x^4 + 3x$ *when* $x \to \pm\infty$ ?

  The leading term is $12x^5$
  As $x \to +\infty, \quad f \to 12(+\infty)^5 = +\infty$
  As $x \to -\infty, \quad f \to 12(-\infty)^5 = -\infty$
- Polynomials can be classified according to their degree (highest power) as **even** or **odd** (note that this may not have same meaning as even and odd regarding symmetry).
  – If a polynomial has an even degree, it goes in the same direction for $x \to \pm\infty$
  – If a polynomial has an odd degree, it goes in opposite directions for $x \to \pm\infty$
  You can use the same test as above to confirm this for yourself