if f = linear, $\Rightarrow$ trapezoidal

$$f(x) = f_0 + x f' + \frac{x^2}{2!} f'' + \frac{x^3}{3!} f''' + \cdots\cdots$$

$$f(x) = f_0 + \frac{f_1 - f_{-1}}{2h} x + \frac{f_1 - 2f_0 + f_{-1}}{2h^2} x^2 + \cdots O(x^3)$$

$$\int_{-h}^{h} f(x)\,dx = \frac{h}{3}\left(f_1 + 4 f_0 + f_{-1}\right) + O(h^5)$$ SIMPSON'S RULE

$\vdots$

$$\int_a^b f(x)\,dx = \frac{h}{3}\Big[f(a) + 4f(a+h) + 2f(a+2h) + 4f(a+3h) + 2f(a+4h) \cdots\cdots 4f(b-h) + f(b)\Big]$$

Keeping higher order terms

$$\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}\left[f_0 + 3f_1 + 3f_2 + f_3\right] + O(h^5)$$

Simpson's $\frac{3}{8}$ rule

$$\int_{x_0}^{x_4} f(x)\,dx = \frac{2h}{45}\left[7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4\right] + O(h^7)$$

Bode's Rule.