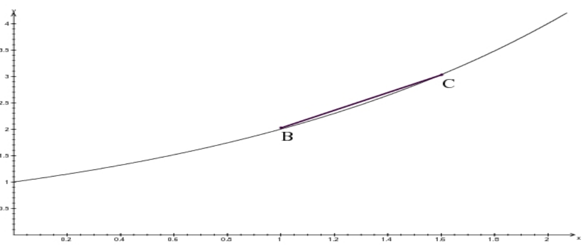
NM/5 NUMERICAL DIFFERENTIATION	**5.1. The forward difference approximation** **5.2. The central difference approximation**
Specification Know how to estimate a derivative using the forward and central difference methods with a suitable value (or sequence of values) of h (c1) Have an empirical and graphical appreciation of the greater accuracy of the central difference formula (c2) Know how to calculate the error in $f(x)$ when there is an error in x (v2)	
Learning objectives Know how to produce forward and central difference approximations to the gradient of a function at a point on its graph. Know how to rearrange the forward difference approximation formula to get a relationship between the error in x and the resultant error in $f(x)$ for a function f.	
p.103-109	Ex.5A all questions **Multiple Choice Test 1**

5.1. The forward difference approximation

Suppose we have a function we cannot differentiate, e.g. $f(x) = 2^x$. How would you find the gradient at a point on the curve $y = f(x)$, say, the point B (1,2)?



Plot the point C to the right of, but near, the point B on the curve.

The gradient of the chord BC is approximately the same as the gradient of the tangent to the curve at B.

This is the basis of "differentiation from first principles".

Suppose C is h to the right of B.

Then the gradient of the chord BC $= \dfrac{\text{difference in } y \text{ co-ords}}{\text{difference in } x \text{ co-ords}} = \dfrac{2^{1+h} - 2^1}{(1+h)-1} = \dfrac{2^{1+h} - 2^1}{h}$

Generalising this, suppose B is the point $(x, 2^x)$ rather than (1, 2). Then

gradient of BC $= \dfrac{2^{x+h} - 2^x}{h}$

and if the curve were $y = f(x)$ rather than $y = 2^x$, gradient of BC $= \dfrac{f(x+h) - f(x)}{h}$.

This is the **forward difference approximation** (FDA) to the gradient of the curve.

E.g. for $f(x) = 2^x$ at (1, 2): we are finding an approximation to $f'(1)$.

$h = 0.1$: $\quad f'(1) \approx \dfrac{f(1.1) - f(1)}{0.1} = \dfrac{2^{1.1} - 2}{0.1} = 1.4355$ (4 d.p.)

$h = 0.05$: $\quad f'(1) \approx \dfrac{f(1.05) - f(1)}{0.05} = \dfrac{2^{1.05} - 2}{0.05} = 1.4106$

$h = 0.025$: $\quad f'(1) \approx \dfrac{f(1.025) - f(1)}{0.025} = \dfrac{2^{1.025} - 2}{0.025} = 1.3984$

$h = 0.0125$: $\quad f'(1) \approx \dfrac{f(1.0125) - f(1)}{0.0125} = \dfrac{2^{1.0125} - 2}{0.0125} = 1.3923$