

14. There are some repeated letters: two Es and two Ds.

$$\text{Case 1: two Es and two Ds} \quad \frac{4!}{2!2!} = 6$$

$$\text{Case 2: either two Es or two Ds} \quad \frac{2}{2} \cdot {}_6C_2 = 2(15) = 30$$

$$\text{Case 3: no repeated letters} \quad {}_7C_4 = 35$$

The only permutation so far is Case 1; therefore, the other cases must be multiplied by the number of orderings of four letters, i.e., $4! = 24$

Total number of four-letter words is:

$$6 + 30(24) + 35(24) = 1566.$$

15. There are 13 possible pairs. Select two: ${}_{13}C_2 = 78$.

There are four cards of each denomination. Select two of each denomination selected above: ${}_4C_2 \cdot {}_4C_2 = 6(6) = 36$.

Select one more card from the remaining 11 denominations: ${}_{11}C_1 = 11$.

The order in which the cards are dealt is unimportant.

Therefore, the total number of ways of being dealt two pairs is $78(36)11 = 311328$.

16. a) Choose one of each: ${}_{10}C_1 \cdot {}_8C_1 = 10(8) = 80$.

b) Choose two of each: ${}_{10}C_2 \cdot {}_8C_2 = 45(28) = 1260$.

c) Same as part (b) except they can switch partners:
 $1260(2) = 2520$

17. To form a rectangle you need two vertical and two horizontal lines.

Selecting two of each: ${}_7C_2 \cdot {}_4C_2 = 21(6) = 126$.

18. a) A chooses six and B gets the rest: ${}_{12}C_6 = 924$ ways.

b) Since unassigned, they can switch. Hence, the subdivision is the same: $924 \div 2 = 462$.

c) A chooses four of 12; B chooses four of eight; C gets the rest:

$${}_{12}C_4 \cdot {}_8C_4 = 495(70) = 34\,650.$$

d) Any one of the three subdivisions in part (c) can be ordered in $3! = 6$ ways. Since unassigned, $34\,650 \div 6 = 5775$ ways.