FUNCTIONS: DOMAIN and RANGE

[Note: In this worksheet, "x" will refer to the *independent* variable of a function and "y" will refer to the *dependent* variable. In reality, any letter could stand for either variable.]

Mathematical functions operate a little like computers – you put something in at one end (a value for the "independent" variable) and you get something out at the other (a value for the "dependent" variable). The restriction for something to be a function is that the input value never yields more than one output value.

For example, for the function $y = f(x) = x + 2$, if I pick a value of 3 for x, then $y = 5$, and I get the ordered pair (3, 5). For any x value I put in, I get only one y value out – y is a function of x.

The terms "domain" and "range" both refer to the sets of values that are *possible* for a function's variables to have. Domain is the set of possible values of the independent variable and range is the set of possible values of the dependent variable. What does "possible" mean? For domain, it means real numbers that yield real numbers as function output. In real-world problems, domain values also have to make sense – you can't have a negative area or length or weight. Range is a little trickier – we'll get back to that.

For the function $y = f(x) = x + 2$, x can be any value, from negative infinity to positive infinity, and we would say the domain of this function is all real numbers ($\mathbb{R}$), or $(-\infty, \infty)$ in interval notation, or $\{x \mid -\infty < x < \infty\}$ in set builder notation (read as "all x such that x is between negative infinity and positive infinity"). All real numbers make sense and yield real results.

So, when can x not be just any value? Here is THE question you have to answer in order to define domain: **are there any x values I *must exclude* for some reason**?

Whatever gets excluded becomes in effect the definition of the domain for that function.

If I can't use a value of 5 for x, but I can use all other values, then $x \neq 5$ defines the domain.

If x has to be greater than 0, then $x > 0$ defines the domain.

Now here's the easy part – there are only two questions you have to ask in order to define domain:

#1 **Rational functions**: are there any x values that will result in a division by zero and therefore make the function undefined?

#2 **Radicals**: are there any x values that will result in a negative number under an even root radical sign (square root, fourth root, sixth root, etc.), yielding a non-real output?

If the answer is "no" to both these questions for a particular function, and there are no real-world constraints, then the domain is all real numbers – done!