find $\int \frac{5+x}{(1-x)(5+x^2)}dx$

using partial fractions

$$\frac{5+x}{(1-x)(5+x^2)} \equiv \frac{A}{x+1} + \frac{Bx+C}{5+x^2}$$

$$\equiv \frac{A(5+x^2)+(Bx+C)(1-x)}{(1-x)(5+x^2)}$$

$$5+x = A(5+x^2)+(Bx+C)(1-x)$$

$$5+x = 5A + Ax^2 + Bx + 1 - Bx^2 - cx \qquad *$$

making $x = 1$

$$5+1 = A(5+1) + \qquad 0$$

$$6 = 6A, \qquad \underline{A = 1}$$

making $x = 0$

$$5 = 5A + C, \qquad 5 = 5 + C, \qquad \underline{C = 0}$$

equating coefficients of $x^2$ *

$$0 = A - B, \qquad \underline{B = A = 1}$$

$$\Rightarrow \qquad \frac{5+x}{(1-x)(5+x^2)} \equiv \frac{1}{1-x} + \frac{x}{5+x^2}$$

$$\int \frac{5+x}{(1-x)(5+x^2)}dx = \int \left(\frac{1}{1-x} - \frac{x}{5+x^2}\right)dx$$

$$\int \frac{5+x}{(1-x)(5+x^2)}dx = -\ln(x-1) + \frac{1}{2}\ln(5+x^2)$$

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