You could actually simplify the formulas furthur more if you want to, and that would produce even more formulas to caculate our purpose:

① $(n+1) \times \frac{n}{2}$ *

$= n+1 \times (\frac{n}{2} + \frac{1}{2}) - \frac{n+1}{2}$

$= n+1 \times (\frac{n+1}{2}) - \frac{n+1}{2}$

$= \boxed{\frac{(n+1)^2}{2} - \frac{n+1}{2}}$ ③

$\frac{1}{2} \times (n+1)$

$= \frac{n+1}{2}$

so adding $\frac{n+1}{2}$ and then subtracting $\frac{n+1}{2}$ sustains the equality of the equation

② $n \times \frac{(n-1)}{2} + n$

$= n \times (\frac{n}{2} - \frac{1}{2}) + n$

$= n \times (\frac{n}{2} - \frac{1}{2} + 1)$ ← It's telling us there are $(\frac{n}{2} - \frac{1}{2})$ times of n, so with 1 more n to add, there's one more time of n

$= n \times (\frac{n}{2} + \frac{1}{2})$

$= \boxed{n \times \frac{n+1}{2}}$ ④

$= \frac{n \times (n+1)}{2}$

$= \frac{n}{2} \times (n+1)$ *

✓ great algebra skills!

* From these two exact same formulas, we could see the relationship and equality between formula ① and formula ②.

At last, I've check formula ①, ②, ③, and ④, and they are all equal, meaning they all result in the same number when their variables (n) represent the some natural number. These four ways of calculating the sum of the first n consecutive numbers are all always valid and related to each other.

Sam, in formula ② you started with

$n \cdot (\frac{n-1}{2}) + n$

(which is what you got in your discussion of n being odd)

and then you derived from it $\frac{n}{2}(n+1)$

(which is what you showed for n is even).

This is sufficient to show that ① and ② are equivalent