2.11 Schrödinger equation

In coordinate representation

$$\hbar i \frac{\partial \Psi}{\partial t} = \int d^3x' \langle x|H|x'\rangle \Psi(x')$$

typical Hamiltonian

$$H = \frac{\vec{p}^2}{2m} + V(x)$$

$$\langle x| \frac{\vec{p}^2}{2m} |x'\rangle = -\frac{\hbar^2}{2m} \partial_{\vec{x}}^2 \, \delta^3(\vec{x}-\vec{x}')$$

$$\langle x| V(x) |x'\rangle = V(x)\, \delta^3(\vec{x}-\vec{x}')$$

$$\int d^3x' \sum_{k=1}^{3} \frac{\partial^2}{\partial x'^2_k} \delta^3(\vec{x}-\vec{x}')\, \Psi(x')$$

$$= \int d^3x' \, \delta^3(x-x') \sum_{k=1}^{3} \frac{\partial^2}{\partial x'^2_k} \Psi(x')$$

↑ partial integrate twice

$$= \vec{\nabla}^2 \Psi$$

$$\Rightarrow \hbar i \frac{\partial \Psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \vec{\nabla}^2 + V(x)\right)\Psi$$

This is the standard Schrödinger equation

Wave mechanics