

Domain and Range Worksheet

Given a function $y = f(x)$, the **Domain** of the function is the set of permissible inputs and the **Range** is the set of resulting outputs. Domains can be found algebraically; ranges are often found algebraically and graphically. Domains and Ranges are sets. Therefore, you must use proper set notation.

When finding the domain of a function, ask yourself what values can't be used. Your domain is everything else. There are simple basic rules to consider:

- The domain of all polynomial functions and exponential functions is the Real numbers **R**.
- Square root functions can not contain a negative underneath the radical. Set the expression under the radical greater than or equal to zero and solve for the variable. This will be your domain.
- Rational functions can not have zeros in the denominator. Determine which values of the input cause the denominator to equal zero, and set your domain to be everything else.
- Log functions must have a positive value in the argument position. Solve for the domain like you would for square root functions.

Examples: Consider $f(x) = x^3 - 6x^2 + 5x - 11$, $g(t) = \sqrt{2 - 3t}$, $h(p) = \frac{p-1}{p^2-4}$.

Answers:

- Since $f(x)$ is a polynomial, the domain of $f(x)$ is **R**.
- Since $g(t)$ is a square root, set the expression under the radical to greater than or equal to zero: $2 - 3t \geq 0 \rightarrow 2 \geq 3t \rightarrow 2/3 \geq t$. Therefore, the domain of $g(t) = \{ t \mid t \leq 2/3 \}$. Confirm by graphing: you will see that the graph "lives" to the left of $2/3$ on the horizontal axis.
- Since $h(p)$ is a rational function, the bottom can not equal zero. Set $p^2 - 4 = 0$ and solve: $p^2 - 4 = 0 \rightarrow (p + 2)(p - 2) = 0 \rightarrow p = -2$ or $p = 2$. These two p values need to be avoided, so the domain of $h(p)$ is $\{ p \mid \text{all } \mathbf{R} \text{ except } p = -2 \text{ or } 2 \}$.

Comment on interval notation:

- The set of all reals can be abbreviated **R**, but not $\{\mathbf{R}\}$. It can also be written $\{(-\infty, \infty)\}$ but not $\{[-\infty, \infty]\}$. It can also be written $\{-\infty < x < \infty\}$.
- For $h(p)$, the domain could be written any of the following ways: $\{(-\infty, -2) \cup (-2, 2) \cup (2, \infty)\}$ or $\{\mathbf{R} \setminus \{-2, 2\}\}$. The backslash \setminus is read as "except".

Whatever method you use, be consistent and correct.