

It is possible to determine the number of solutions of a system of linear equations by using the row echelon form of the augmented matrix of the system.

10.1.1 The Row Echelon Form

The row echelon form of a matrix is a matrix in which the leading ones are in the main diagonal and the entries below the leading ones are zero. The row echelon form of a matrix is unique.

Example: Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. This matrix is in row echelon form.

Example: Consider the matrix $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. This matrix is not in row echelon form.

Example: Consider the matrix $C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. This matrix is in row echelon form.

Example: Consider the matrix $D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. This matrix is in row echelon form.

Example: Consider the matrix $E = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. This matrix is in row echelon form.

Example: Consider the matrix $F = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. This matrix is in row echelon form.

10.1.2 The Rank of a Matrix

10.1.2.1 Rank of a Matrix

The rank of a matrix is the number of linearly independent rows (or columns) of the matrix. The rank of a matrix is equal to the number of non-zero rows in its row echelon form.

10.1.2.2 Rank of a Matrix

Example: Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of A is 2.

Example: Consider the matrix $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of B is 3.

Example: Consider the matrix $C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of C is 2.

Example: Consider the matrix $D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of D is 3.

Example: Consider the matrix $E = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of E is 2.

Example: Consider the matrix $F = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of F is 3.

10.1.2.3 Rank of a Matrix

The rank of a matrix is the number of linearly independent rows (or columns) of the matrix. The rank of a matrix is equal to the number of non-zero rows in its row echelon form.

10.1.2.4 Rank of a Matrix

Example: Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of A is 2.

Example: Consider the matrix $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of B is 3.

Example: Consider the matrix $C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of C is 2.

Example: Consider the matrix $D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of D is 3.

Example: Consider the matrix $E = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of E is 2.

Example: Consider the matrix $F = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of F is 3.

Example: Consider the matrix $G = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of G is 2.

Example: Consider the matrix $H = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of H is 3.

Example: Consider the matrix $I = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of I is 2.

Example: Consider the matrix $J = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of J is 3.

Example: Consider the matrix $K = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$. The rank of K is 2.

Example: Consider the matrix $L = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$. The rank of L is 3.