

$$\begin{aligned}\frac{1}{(x-2)^2(x+3)} &= \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x+3)} \\ &= \frac{A(x-2)(x+3) + B(x+3) + C(x-2)^2}{(x-2)^2(x+3)}\end{aligned}$$

$$1 \equiv A(x-2)(x+3) + B(x+3) + C(x-2)^2$$

$$1 \equiv A(x^2 + x - 6) + B(x+3) + C(x^2 - 4x + 4) \quad *$$

putting $x = 2$

$$\begin{aligned}1 &\equiv 0 + B(2+3) + 0 \\ 1 &\equiv 5B, \quad \therefore B = \frac{1}{5}\end{aligned}$$

putting $x = -3$

$$\begin{aligned}1 &\equiv 0 + 0 + C(-5)^2 \\ 1 &\equiv 25C, \quad \therefore C = \frac{1}{25}\end{aligned}$$

equating coefficients of x^2 from *

$$0 = A + C, \quad A = -C, \quad \therefore A = -\frac{1}{25}$$

$$\frac{1}{(x-2)^2(x+3)} = -\frac{1}{25(x-2)} + \frac{1}{5(x-2)^2} + \frac{1}{25(x+3)}$$