

$$(7) \quad \psi(x) \rightarrow \psi(x) + \epsilon \varphi(x) \quad \left\{ \text{A VARIATION } \varphi(x) \text{ IS ADDED} \right.$$

$$(8) \quad \frac{\partial}{\partial \epsilon} (\Delta x)^2 (\Delta p)^2 = (\Delta p)^2 \frac{\partial}{\partial \epsilon} (\Delta x)^2 + (\Delta x)^2 \frac{\partial}{\partial \epsilon} (\Delta p)^2 = 0$$

$$(9) \quad (\Delta x)^2 \left[ \left( \frac{\hbar}{4\pi} \right)^2 \left( \frac{1}{\Delta x} \right)^4 \frac{\partial}{\partial \epsilon} (\Delta x)^2 + \frac{\partial}{\partial \epsilon} (\Delta p)^2 \right] = 0 \quad \left\{ \begin{array}{l} \text{UNDETERMINED } \Delta p \\ \text{REPLACED BY (1)} \end{array} \right.$$

$$(10) \quad \frac{\partial}{\partial \epsilon} \left[ - \left( \frac{\hbar}{4\pi} \right)^2 \left( \frac{1}{\Delta x} \right)^2 + (\Delta p)^2 \right] = 0 \quad \left\{ (\Delta x)^2 > 0, \text{ OTHERWISE } \begin{array}{l} (\Delta p)^2 \rightarrow \infty \\ \epsilon \rightarrow \infty \end{array} \right.$$

$$(11) \quad \frac{\partial}{\partial \epsilon} \left[ - \left( \frac{\hbar}{4\pi} \right)^2 \int \left( \frac{d\psi(x)}{dx} \right)^2 dx + 2m \int (E - V(x)) \psi^2(x) dx \right] = 0 \quad \left\{ \begin{array}{l} \text{USING} \\ (5) \text{ \& } (6) \end{array} \right.$$

$$(12) \quad - \left( \frac{\hbar}{2\pi} \right)^2 \int \frac{d\psi(x)}{dx} \cdot \frac{d\varphi(x)}{dx} dx + 2m \int (E - V(x)) \psi(x) \varphi(x) dx = 0 \quad \left\{ \frac{\partial}{\partial \epsilon} \text{ \& } \epsilon = 0 \text{ (7)} \right.$$

$$(13) \quad \int \left[ \left( \frac{\hbar}{2\pi} \right)^2 \frac{d^2\psi(x)}{dx^2} + 2m(E - V(x))\psi(x) \right] \varphi(x) dx = 0 \quad \left\{ \begin{array}{l} \text{INTEGRATION BY PARTS} \\ \varphi(x) = 0 \text{ AT BORDERS} \end{array} \right.$$

$$(14) \quad \boxed{\frac{d^2\psi(x)}{dx^2} + 2m \left( \frac{2\pi}{\hbar} \right)^2 (E - V(x)) \psi(x) = 0} \quad \left\{ (13) = 0 \text{ FOR ALL VARIATIONS } \varphi(x) \right.$$

SCHRÖDINGER'S WAVE EQUATION