Keynesian Supermultiplier

National income accounting equation:

$$Y = C + I + G + X - M \tag{1}$$

Consumption function includes autonomous consumption, a, plus the marginal propensity to consume, b, times disposable income:

$$C = a + bY_d \tag{2}$$

Disposable income is aggregate national income less the tax bill, T:

$$Y_d = Y - T \tag{3}$$

Assuming a flat rate income tax, t, the tax bill is equal to the tax rate times aggregate national income:

$$T = tY (4)$$

Substituting equation 4 into equation 3 gives:

$$Yd = Y - tY \tag{5}$$

Investment may be thought of as having an autonomous component, d, but also a component that is a function of income (not disposable income). So we can introduce the marginal propensity to invest, e:

$$I = d + eY \tag{6}$$

Government spending remains autonomous:

$$G = G \tag{7}$$

Aggregate exports are also exogenous, since it is determined by other nations' demand:

$$X = X$$
 (8)

Imports, however, may be thought of as having an autonomous component, f, and a component that depends on disposable income (marginal propensity to import, m):

$$M = f + mY_d \tag{9}$$

Substituting equations 2, 6, and 9 into equation 1 gives:

$$Y = a + bY_d + d + eY + G + X - (f + mY_d)$$
 (10)

Substituting equation 5 into equation 10 gives:

$$Y = a + b(Y - tY) + d + eY + G + X - f - m(Y - tY)$$
(11)

Multiplying through (watch those signs!):

$$Y = a + bY - btY + d + eY + G + X - f - mY + mtY$$
 (12)