$$0 < \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$$

$$= \int_0^1 \frac{x^4 - 4x^5 + 6x^6 - 4x^7 + x^8}{1+x^2}\,dx \quad \text{(expansion of terms in the numerator)}$$

$$= \int_0^1 \left(x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2}\right) dx$$

(polynomial long division)

$$= \left(\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\arctan x\right)\Bigg|_0^1 \quad \text{(definite integration)}$$

$$= \frac{1}{7} - \frac{2}{3} + 1 - \frac{4}{3} + 4 - \pi \quad \text{(since } \arctan(1) = \pi/4 \text{ and } \arctan(0) = 0\text{)}$$

$$= \frac{22}{7} - \pi. \quad \text{(addition)}$$