

(52)

2.11 Schrödinger equation

In coordinate representation

$$i\hbar \frac{\partial \psi}{\partial t} = \int d^3x' \langle x | H | x' \rangle \psi(x')$$

typical Hamiltonian

$$H = \frac{\vec{p}^2}{2m} + V(x)$$

$$\langle x | \frac{\vec{p}^2}{2m} | x' \rangle = -\frac{\hbar^2}{2m} \nabla_x^2 \delta^3(\vec{x} - \vec{x}')$$

$$\langle x | V(x) | x' \rangle = V(x) \delta^3(\vec{x} - \vec{x}')$$

$$\int d^3x' \sum_{k=1}^3 \frac{\partial^2}{\partial x_k'^2} \delta^3(\vec{x} - \vec{x}') \psi(x')$$

$$= \int d^3x' \delta^3(\vec{x} - \vec{x}') \sum_{k=1}^3 \frac{\partial^2}{\partial x_k'^2} \psi(x')$$

↑ partial integrate twice

$$= \nabla^2 \psi$$

$$\Rightarrow i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V(x) \right) \psi$$

This is the standard Schrödinger equation

Wave mechanics