Conic Sections:

The Hyperbola

The equation for a hyperbola has both an x^2 and y^2 term, with one of them being added and the other subtracted. Once the equation is in standard form, which one is subtracted (x^2 or y^2) determines whether the hyperbola is "horizontal" or "vertical".

FORMULA FOR HYPERBOLAS

Once the formula for the hyperbola is in standard form (described below), *a* is always in the denominator of the term that's added, and *b* is always in the denominator of the term that's subtracted.

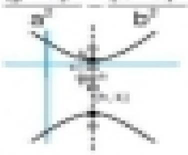
Horizontal Transverse Axis:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

centre: (h, k)
vertices: $(h + a, k)$, $(h - a, k)$
foci: $(h + c, k)$, $(h - c, k)$, where $c^2 = a^2 + b^2$
asymptotes: $y - k = \pm\frac{b}{a}(x - h)$

Vertical Transverse Axis:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



centre: (h, k)
vertices: $(h, k + a)$, $(h, k - a)$
foci: $(h, k + c)$, $(h, k - c)$, where $c^2 = a^2 + b^2$
asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

Example 1: Find the centre, vertices, foci and asymptotes of the hyperbola $x^2 + 8x - y^2 + 10y = 13$.

Solution: First we need to get the equation into the standard form. We start by completing the squares for *x* and for *y*.

$$(x^2 + 8x) - (y^2 - 10y) = 13$$
$$(x^2 + 8x + 16 - 16) - (y^2 - 10y + 25 - 25) = 13$$
$$(x^2 + 8x + 16) - 16 - (y^2 - 10y + 25) + 25 = 13$$
$$(x^2 + 8x + 16) - (y^2 - 10y + 25) = 13 + 16 - 25$$
$$(x + 4)^2 - (y - 5)^2 = 4$$
$$\frac{(x+4)^2}{4} - \frac{(y-5)^2}{4} = 1$$

Now we can see h, k, a and b: $h = -4$, $k = 5$, $a = 2$ and $b = 2$. The *x* term is added, so its