



## Product Rule & Quotient Rule

### PRODUCT RULE

The Product Rule is used to take the derivative of a "product", or in other words the derivative of two functions multiplied together. The general form looks like

$$(f(x)g(x))' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

In other words, take the derivative of the first function multiplied by the second function and add it to the derivative of the second function multiplied by the first function. You can make this a bit easier for yourself by writing out a simple table and then plugging those expressions into the equation above.

$$\begin{array}{l} f(x) = \\ f'(x) = \end{array} \qquad \begin{array}{l} g(x) = \\ g'(x) = \end{array}$$

**Exercise 1:** Find the derivative of  $F(x) = x^2 \sin x$ .

**Solution:** This function is a product of  $x^2$  and  $\sin x$ . Let  $f(x) = x^2$  and  $g(x) = \sin x$ . Make the table:

$$\begin{array}{l} f(x) = x^2 \\ f'(x) = 2x \end{array} \qquad \begin{array}{l} g(x) = \sin x \\ g'(x) = \cos x \end{array}$$

Now plug into the formula:

$$F'(x) = 2x \cdot \sin x + \cos x \cdot x^2$$

The Product Rule is flexible enough that you can do the two halves of the derivative in either order, but if you start with "derivative of the first function" first, it will help you remember the next rule, which is less flexible.

### QUOTIENT RULE

The Quotient Rule is used to take the derivative of a "quotient", two functions being divided. The general form looks like:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

You can make the same list as above and then plug into the formula to find the derivative.

**Example 2:** Find the derivative of  $F(x) = \frac{x^2}{\sin x}$

**Solution:** This function is the quotient of  $x^2$  divided by  $\sin x$ . Let  $f(x) = x^2$  and  $g(x) = \sin x$ . Make the table:

$$\begin{array}{l} f(x) = x^2 \\ f'(x) = 2x \end{array} \qquad \begin{array}{l} g(x) = \sin x \\ g'(x) = \cos x \end{array}$$