

**Worksheet: Exponential and Logarithmic Functions  
Solutions**

**Identities and Equivalent Equations:**

For all of the following the base is a positive real (but not 1):  $a > 0$ ,  $a \neq 1$ .

- Definition of Logarithm:

$$a^u = v \Leftrightarrow \log_a v = u, \quad u \in \mathbb{R}, v > 0 \quad (1)$$

- Equivalent Equations:

$$\log_a u = \log_a v \Leftrightarrow u = v, \quad u, v > 0 \quad (2)$$

$$a^u = a^v \Leftrightarrow u = v, \quad u, v \in \mathbb{R} \quad (3)$$

Remember that rule (2) can create extraneous solutions;  
check your results in the *original* equation.

- Identities:

$$a^{\log_a u} = u, \quad u > 0 \quad (4)$$

$$\log_a a^u = u, \quad u \in \mathbb{R} \quad (5)$$

$$\log_a (uv) = \log_a u + \log_a v, \quad u, v > 0 \quad (6)$$

$$\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v, \quad u, v > 0 \quad (7)$$

$$\log_a u^r = r \log_a u, \quad u > 0, r \in \mathbb{R} \quad (8)$$

$$\log_a u = \frac{\log_b u}{\log_b a}, \quad u, a, b > 0, b \neq 1 \quad (9)$$

**Problems:**

1. Solve

$$\ln(k+5) + \ln(k+2) = \ln(14k)$$

**Solution:**

Using rules (6) and (2) from the list above, we can convert this logarithmic equation into a polynomial equation which we can then solve for  $k$ .

$$\begin{aligned} \ln(k+5) + \ln(k+2) = \ln(14k) &\implies \ln[(k+5)(k+2)] = \ln(14k) \\ &\implies (k+5)(k+2) = 14k \\ &\implies k^2 + 7k + 10 = 14k \\ &\implies k^2 - 7k + 10 = 0 \\ &\implies (k-5)(k-2) = 0 \\ &\implies k \in \{2, 5\}. \end{aligned}$$