SPSU Math 1113: Precalculus
§8.3 Trig Identities

Two functions are said to be **identically equal** if

f(x) = g(x)

for every value of x for which both functions are defined. Such an equations is referred to as an **identity.**

Basic Trig Identities

Quotients:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocals:

$\csc\theta = \frac{1}{\sin\theta}$ $\qquad \sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

Pythagorean:

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$ $\qquad \cot^2\theta + 1 = \csc^2\theta$

Even-Odd:

$\sin(-\theta) = -\sin\theta$ $\qquad \csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\qquad \sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\qquad \cot(-\theta) = -\cot\theta$

Use Standard Algebra to Simplify Trig Expressions

Often simply rewriting in terms of sin & cos is enough.

a. $\frac{\cot\theta}{\csc\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1} = \cos\theta$

b. Show that $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

Multiply numer & denom by *(1 – sin θ)*

$\frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta} = \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta} = \frac{1-\sin\theta}{\cos\theta}$ q.e.d.

c. Simplify $\frac{\sin^2 v-1}{\tan v\sin v-\tan v}$ by factoring

$\frac{\sin^2 v-1}{\tan v\sin v-\tan v} = \frac{(\sin v-1)(\sin v+1)}{\tan v(\sin v-1)} = \frac{\sin v+1}{\tan v}$

14. $\frac{1}{1-\cos v} + \frac{1}{1+\cos v} =$

$= \frac{1}{(1-\cos v)}\frac{(1+\cos v)}{(1+\cos v)} + \frac{1}{(1+\cos v)}\frac{(1-\cos v)}{(1-\cos v)}$

$= \frac{(1+\cos v)+(1-\cos v)}{(1-\cos^2 v)} = \frac{2}{\sin^2 v}$

Establish the Identities

This means prove that the identity is valid.

20. $\sec\theta\cdot\sin\theta = \tan\theta$

$\frac{1}{\cos\theta}\cdot\sin\theta = \frac{\sin\theta}{\cos\theta}$

26. $\sin u\csc u - \cos^2 u = \sin^2 u$

$\sin u\,\frac{1}{\sin u} - \cos^2 u = \sin^2 u$

$1 - \cos^2 u = \sin^2 u$ q.e.d.

30. $(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = 1$

$\csc^2\theta - \cot^2\theta = \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} = \frac{1-\cos^2\theta}{\sin^2\theta}$

$= \frac{\sin^2\theta}{\sin^2\theta} = 1$ q.e.d.

62. $\frac{\sec\theta-\cos\theta}{\sec\theta+\cos\theta} = \frac{\sin^2\theta}{1+\cos^2\theta}$

$\frac{\frac{1}{\cos\theta}-\cos\theta}{\frac{1}{\cos\theta}+\cos\theta} = \frac{1-\cos^2\theta}{1+\cos^2\theta} = \frac{\sin^2\theta}{1+\cos^2\theta}$ q.e.d

68. $\frac{1-\cot^2\theta}{1+\cot^2\theta} + 2\cos^2\theta = 1$

$= \frac{1-\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}} + 2\cos^2\theta = \sin^2\theta - \cos^2\theta + 2\cos^2\theta$

$= \sin^2\theta + \cos^2\theta = 1$ Q.E.D.

Homework

Do problems for §8.3

Read next section