Understanding Properties of Logarithms

While it may be easy for one to state or use any of the properties of logarithms discussed in class, this does not imply that these concepts are understood by the individual. A closer look at each of the logarithm properties may help in figuring out how and why they work.

The Logarithm of a Product Property states that the logarithm of a product is equal to the sum of the logarithms, or symbolically $\log_a(MN)=\log_a M+\log_a N$. The core idea is that a product becomes a sum. This idea should be familiar; recall that a property of exponents is that the product of two exponential expressions with the same base is equal to the sum of the exponents, or symbolically $x^u \cdot x^v = x^{u+v}$. Again, the core idea is that a product becomes a sum. The two statements differ in that with logarithms, the *single* logarithm of the product becomes the sum of *two* logarithms, while with exponents, the product of *two* exponential terms becomes the sum in *one* exponential term. A proof of this property of logarithms is as follows:

$$\log_a M+\log_a N=x \tag{1}$$

solving for each logarithm gives:

$$\log_a M=x-\log_a N \qquad \log_a N=x-\log_a M \tag{2}$$

rewriting each logarithm as an exponent gives:

$$a^{x-\log_a N}=M \qquad a^{x-\log_a M}=N \tag{3}$$

multiplying each side by equal values gives:

$$a^{x-\log_a N}\cdot a^{x-\log_a M}=M\cdot N \tag{4}$$

applying the product of exponents with the same base rule we obtain:

$$a^{x-\log_a N+x-\log_a M}=MN \tag{5}$$

collecting like terms and factoring in the exponent we obtain:

$$a^{2x-(\log_a M+\log_a N)}=MN \tag{6}$$

substituting equation (1) back in gives:

$$a^{2x-x}=MN \tag{7}$$

collecting like terms gives:

$$a^x=MN \tag{8}$$

rewriting as a logarithm gives:

$$\log_a(MN)=x \tag{9}$$

which proves:

$$\log_a M+\log_a N=\log_a(MN) \tag{10}$$