

Resonance

Resonance occurs when an external force is applied to a system at a frequency equal to the natural frequency of the system.

Example:

A mass-spring system has a mass of 2 kg and a spring constant of 100 N/m.

The external force is $F(t) = 10 \cos(5t)$ N.

The initial conditions are $x(0) = 0$ and $\dot{x}(0) = 0$.

Find the displacement $x(t)$.

Solve the equation:

$m\ddot{x} + kx = F(t)$

For $m = 2$ kg, $k = 100$ N/m, and $F(t) = 10 \cos(5t)$ N, the equation becomes:

Resonance condition:

Resonance occurs when the frequency of the external force matches the natural frequency of the system. In this case, the natural frequency is $\omega_0 = \sqrt{k/m} = \sqrt{100/2} = 5$ rad/s, which matches the frequency of the external force $\omega = 5$ rad/s. This results in unbounded growth of the displacement over time, a phenomenon known as resonance. Thus, the displacement $x(t)$ will grow without bound as $t \rightarrow \infty$.

Transient response versus steady-state response:

The transient response is the initial behavior of the system, which decays over time. The steady-state response is the long-term behavior of the system, which is a sinusoidal function with a constant amplitude.

Resonance:

Resonance occurs when the frequency of the external force matches the natural frequency of the system.

Resonance leads to unbounded growth of the displacement.

2.2.2. AC circuits and resonance