$$\frac{d(\cosec\ x)}{dx} = \frac{d\left(\dfrac{1}{\sin x}\right)}{dx} = \frac{d(\sin x)^{-1}}{dx}$$

let $\quad y = (\sin x)^{-1}, \quad t = \sin x$

then $\quad y = t^{-1}$

$$\frac{dy}{dt} = (-1)(t^{-2}) \qquad\qquad \frac{dt}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = -t^{-2}\cdot\cos x = -(\sin x)^{-2}\cdot\cos x$$

$$\frac{dy}{dx} = -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x.\sin x} = -\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$$

$$\frac{d(\cosec\ x)}{dx} = -\cotan\ x.\cosec\ x$$