$$\frac{d(\operatorname{cosec} x)}{dx} = \frac{d\left(\frac{1}{\sin x}\right)}{dx} = \frac{d(\sin x)^{-1}}{dx}$$

let $y = (\sin x)^{-1}$, $t = \sin x$

then $y = t^{-1}$

$$\frac{dy}{dt} = (-1)(t^{-2}) \qquad \frac{dt}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -t^{-2}.\cos x = -(\sin x)^{-2}.\cos x$$

$$\frac{dy}{dx} = -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x.\sin x} = -\frac{\cos x}{\sin x}.\frac{1}{\sin x}$$

$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cotan} x.\operatorname{cosec} x$$