

Table A1
Common theoretical probability distributions based on MM or LM parameter estimation included in the DFET

Theoretical probability distribution	Probability density function for random variable $F(x)$	Description/assumptions/limitations
Normal distribution (N/MM) (Stedinger et al., 1993; Alexander, 2001)	$F(x) = \frac{1}{\sqrt{2\pi s^2}} \exp\left[-\frac{1}{2}\left(\frac{x-\bar{x}}{s}\right)^2\right]$ <p>where: s = standard deviation of observed values x = observed value \bar{x} = mean of observed values</p>	<ul style="list-style-type: none"> • Only used in hydrology to describe well-behaved phenomena, e.g., average annual streamflow (continuous and independent variables) • Distribution is symmetrical about the mean with skewness coefficient equal or close to zero; therefore limited application in flood hydrology • Generation of negative flows can occur when the minima of data sets are examined
Log-Normal (LN/MM) (Yevjevich, 1982)	$F(x) = \frac{1}{x\sqrt{2\pi s_y^2}} \exp\left[-\frac{1}{2}\left(\frac{\log(x) - \overline{\log(x)}}{s_y}\right)^2\right]$ <p>where: N = total number of observations s_y = standard deviation of the observed value logarithms $\overline{\log(x)}$ = mean of observed value logarithms x = observed value</p>	<ul style="list-style-type: none"> • Normal distribution based on the observed value logarithms with a near-symmetrical distribution or skewness coefficient close to zero, therefore limited application in flood hydrology • The \log_{10}-transformation of data tends to reduce positive skewness commonly found in hydrology
Log-Pearson Type 3 (LP3/MM) (Chow et al., 1988)	$F(x) = \frac{\lambda^\beta (\log(x) - \mathcal{E})^{\beta-1} e^{-\lambda(\log(x) - \mathcal{E})}}{x\Gamma(\beta)}$ <p>where: $\beta = \left(\frac{2}{g(\log(x))}\right)^2$ g = skewness coefficient Γ = gamma function $\lambda = \frac{s_y}{\sqrt{\beta}}$ \mathcal{E} = lower bound, $\overline{\log(x)} - s_y\sqrt{\beta}$ s_y = standard deviation of the observed value logarithms $\overline{\log(x)}$ = mean of observed value logarithms x = observed value</p>	<ul style="list-style-type: none"> • Common form of the Pearson Type 3 distribution used in hydrological analyses and represents the distribution of the observed value logarithms • Three-parameter Gamma distribution with a third parameter (lower bound, i.e., mean displayed by a constant from the origin) introduced • Includes the LN distribution as a special case when the skewness equals zero • Fit most sets of hydrological data in South Africa and is the standard distribution for frequency analysis in the USA
General Extreme Value (GEV/MM) (Alexander, 2001)	$F(x) = \exp\left[-\left(1 - k\frac{x-\mu}{\alpha}\right)^{\frac{1}{k}}\right]$ <p>where: α = positive scale parameter k = shape parameter μ = location parameter x = observed value</p>	<ul style="list-style-type: none"> • GEV distributions are used in cases where the tail of the distribution of hydrological events decays exponentially within a hydrological year • Family of 3 sub-types of distributions which are classified according to the value of the skewness coefficient (g) or shape parameter (k): <ul style="list-style-type: none"> ○ Extreme Value Type 1 (EV1)/Gumbel distribution ($g = 1.14$ or $k = 0$) ○ Limited application in flood hydrology ○ Extreme Value Type 2 (EV2)/Fréchet distribution ($g > 1.14$ and $k < 0$) Natural data characterised by an EV2 distribution will have \log_{10}-transformed data which are EV1 distributed ○ Extreme Value Type 3 (EV3)/Weibull distribution ($g < 1.14$ and $k > 0$)
Generalised Logistic (GLO/LM) (Kjeldsen and Jones, 2004; Gupta and Kundu, 2007)	$F(x) = \frac{e^{-\frac{x-\mu}{\alpha}}}{\alpha\left(1 + e^{-\frac{x-\mu}{\alpha}}\right)^2}$ <p>where: α = scale parameter μ = location parameter x = observed value</p>	<ul style="list-style-type: none"> • Standard method for flood frequency analysis in the UK • Two generalisations of the GLO distribution are available: <ul style="list-style-type: none"> ○ Skew logistic and proportional reversed hazard logistic (PRHL) distributions ○ Three-parameter distribution with location, scale and skewness parameters ○ Skewness can either be positive or negative with a probability density function (PDF) which is uni-modal and log-concave in nature ○ The distribution function, hazard function and different moments of the skew logistic distribution cannot be obtained in explicit forms and are therefore difficult to use in practice, while the PRHL distribution has distribution and hazard functions with explicit forms and the moments can be expressed in terms of digamma and/or polygamma functions