$$\frac{\sin^2\alpha + 2\cos\alpha - 1}{\sin^2\alpha + 3\cos\alpha - 3} = \frac{1}{1 - \sec\alpha}$$

$$\frac{\sin^2\alpha + 2\cos\alpha - 1}{\sin^2\alpha + 3\cos\alpha - 3} = \frac{1}{1 - \sec\alpha}$$ identity to be verified

$$\frac{\left(1 - \cos^2\alpha\right) + 2\cos\alpha - 1}{\left(1 - \cos^2\alpha\right) + 3\cos\alpha - 3} = \frac{1}{1 - \sec\alpha}$$ Pythagorean identity

$$\frac{-\left(\cos^2\alpha - 2\cos\alpha\right)}{-\left(\cos^2\alpha - 3\cos\alpha + 2\right)} = \frac{1}{1 - \sec\alpha}$$ combining term

$$\frac{(\cos\alpha)(\cos\alpha - 2)}{(\cos\alpha - 1)(\cos\alpha - 2)} = \frac{1}{1 - \sec\alpha}$$ factoring

$$\frac{\cos\alpha}{\cos\alpha - 1} = \frac{1}{1 - \sec\alpha}$$ algebraic manipulation

$$\frac{\left(\frac{\cos\alpha}{\cos\alpha}\right)}{\left(\frac{\cos\alpha - 1}{\cos\alpha}\right)} = \frac{1}{1 - \sec\alpha}$$ algebraic manipulation

$$\frac{1}{1 - \frac{1}{\cos\alpha}} = \frac{1}{1 - \sec\alpha}$$ algebraic manipulation

$$\frac{1}{1 - \sec\alpha} = \frac{1}{1 - \sec\alpha}$$ reciprocal identity