WORKSHEET 1

Addition, multiplication, and exponents.

In this class we will assume as little as possible. In fact, a little bit of incorrect knowledge is much more dangerous than no knowledge at all. So do your best to forget EVERYTHING, and we'll start from scratch. Well, maybe not quite everything. In all mathematics, you have to start by assuming *something*. From these basic assumptions everything else can be derived. Here we will assume that you know what two things are: The numbers 1, 2, 3, etc.., and the fact that you can count. "To count" means that for any number you can think of, there is a "next number." That is, each number has a "successor."

In this worksheet, we will begin to develop some of the basic operations of arithmetic by repetition. So far all we know how to do is count, so that's all we can possibly repeat. But this turns out to be very useful!

Question 1. If we begin with a number a, and repeat our counting "operation" b times, then we end up at a new number, c. Write c in terms of a and b. This is how we define "addition."

Question 2. If we add a number a it to itself b times, then we end up at a new number, c. Write c in terms of a and b. This is how we define "multiplication."

Question 3. If we multiply a number a by itself b times, then we end up at a new number, c. Write c in terms of a and b. This is how we define "exponentiation."

Whenever we have an operation, it will be helpful if we can list all of its useful properties. For example, an operation $\wedge$ is said to be *commutative* if

$$a \wedge b = b \wedge a$$

for all a and b.

Question 4. Are addition, multiplication, and exponentiation commutative? That is, are each of the following statements true, for <u>all</u> values of a and b? For those that are not, find counter-examples. (i.e. numbers a and b so that $a \wedge b$ is not equal to $b \wedge a$.)