MATH 1310
Integral Calculus with Applications
Winter 2009

April 1

<u>Worksheet 2</u>

1. (a) Use the method of slicing to find the volume of a pyramid on a square base of side 2cm with altitude 1cm, where the peak of the pyramid is directly above the centre of the base.

   (b) Use the method of slicing to the find the volume of a (highly!) skewed pyramid on a square base of side 2cm with altitude 1cm, where the peak of the pyramid is not directly over the centre of the base but rather above a point 1km to the left of the centre of the base.

   (c) How do the two volumes computed above compare?

2. Let r and R be two positive numbers, where $0 < r < R$. If the disk of radius r centered at the point $(R, 0)$ is revolved about the y-axis the resulting volume swept out is a doughnut. Find its volume.

3. Let $A(r)$ denote the area of a disk of radius r, and $C(r)$ the circumference of a circle of radius r. It turns out that $\frac{d}{dr}A(r) = C(r)$. Explain this fact using a Riemann sum argument. (This is analogous to our justification of the method of slicing to compute the volume of a solid.)

4. Consider the indefinite integral $\int \sec^m x \tan^n x \, dx$.

   (a) Show that in the case $m = 2$ and $n = 3$, the integral can be solved by means of the substitution $u = \sec x$. Can you find general conditions on m and n such that this same substitution will work?

   (b) Show that if $m = 4$ and $n = 1$, then the integral can be solved by means of the substitution $u = \tan x$. What are the most general conditions on m and n for which this same substitution will work?