L3.1 The symbolic solution of $y' = -2xy$, $y(0) = 2$ is $y = 2e^{-x^2}$. Derive it and display a full answer check

<u>Solution</u>

$y' + 2xy = 0$ std form $y' + py = r$

$\dfrac{(Wy)'}{W} = 0$ $W =$ integrating factor $e^{\int p\,dx}$

$W = e^{x^2}$

$(Wy)' = 0$

$Wy = c$ Quadrature

$y = c\,e^{-x^2}$ $1/W = e^{-x^2}$

$2 = c\,e^{0}$ use $y(0) = 2$

$\boxed{y = 2e^{-x^2}}$ candidate solution

<u>Answer check</u>.

$$
\begin{aligned}
\text{LHS} &= y' \\
&= (2e^{-x^2})' \\
&= 2(-2x)e^{-x^2} && \text{use } (e^u)' = u'e^u \\
&= (-2x)\,2e^{-x^2} \\
&= (-2x)\,y \\
&= \text{RHS} && \text{DE verified}
\end{aligned}
$$

$$
\begin{aligned}
\text{LHS} &= y(0) \\
&= 2e^{-x^2}\Big|_{x=0} \\
&= 2e^{0} \\
&= 2 \\
&= \text{RHS} && \text{IC verified}
\end{aligned}
$$