Solutions to Polynomial Functions, Zeros, and Factoring Worksheet

Real zeros:

One	none	two

Imaginary zeros:

Two	six	two

We know this because the degree tells us how many zeros the function will have, the graph shows us the real zeros, so we find the difference between the two to find how many imaginary zeros each function has.

Completely factor the polynomial functions:

1. $f(x)=2x^4+x^3-19x^2-9x+9$

   Looking at the graph of this function, it appears that -3, -1, ½, and 3 are zeros. We will test some with synthetic division, and verify the last two with the quadratic formula or factoring.

-3	2	1	-19	-9	9
	↓	-6	15	12	-9
	2	-5	-4	3	0

   Now repeat with -1 and the quotient row:

-1	2	-5	-4	3
	↓	-2	7	-3
	2	-7	3	0

   Since the remainder is 0 for both -3 and -1, both of these are zeros of this function.

   The quotient row gives us a quadratic function, $2x^2-7x+3$, which we set equal to zero and solve for x (by factoring or by the quadratic formula). Factoring gives $(2x-1)(x-3)=0$, so setting each factor equal to zero and solving, we get the last two zeros are ½ and 3, as we thought from the graph.

   Zeros: -3, -1, ½, and 3; so factors are $x+3,\ x+1,\ x-\frac{1}{2},\ x-3$. The leading coefficient of the polynomial is 2, so the factored form is $f(x)=2(x+3)(x+1)(x-\frac{1}{2})(x-3)$.

2. $f(x)=x^3+2x^2-3x-6$

   Looking at the graph of this function, it appears that -2 is a zero, and that there are two more zeros that are not integers. Since the leading coefficient is one, these two zeros cannot be rational, so we will have to find them from the quadratic formula. We test the -2 by synthetic division:

-2	1	2	-3	-6
	↓	-2	0	6
	1	0	-3	0

   Since the remainder is 0 for -2, -2 is a zero of this function.

   The quotient row gives us a quadratic function, x^2-3, which we set equal to zero and solve for x (by the square-root method or by the quadratic formula). The square root method gives the following solutions (which the quadratic formula also gives):

$$x^2-3=0$$
$$x^2=3$$
$$x=\pm\sqrt{3}$$

   Thus the two remaining zeros are irrational, as we suspected from the graph.

   Zeros: -2, $\sqrt{3}$, $-\sqrt{3}$; so factors are $x+2,\ x-\sqrt{3},\ x+\sqrt{3}$. The leading coefficient is 1, so the factored form is $f(x)=(x+2)(x-\sqrt{3})(x+\sqrt{3})$.