Then I'll simplify the expressions in the numerator and denominator of the first term a bit:

$$\frac{[1 + \sin 2x - (1 - \sin 2x)](\cos 2x)^2}{1 - (\sin 2x)^2} - \frac{1}{\sin 2x}$$

and then a little more:

$$\frac{(2 \sin 2x)(\cos 2x)^2}{(\cos 2x)^2} - \frac{1}{\sin 2x}$$

(Here I used the identity $(\sin x)^2 + (\cos x)^2 = 1$)

and some more

$$2 \sin 2x - \frac{1}{\sin 2x}$$

Now I have something that really looks good. I'll put everything over a common denominator and subtract:

$$\frac{2(\sin 2x)^2 - 1}{\sin 2x}$$

Looking at this, I'd say that this looks similar to something in the identity:

$$\cos 2k = 1 - 2(\sin k)^2$$

If I put 2x in everywhere there's an k, and multiply by -1, here's what I get:

$$-\cos 4x = 2(\sin 2x)^2 - 1$$