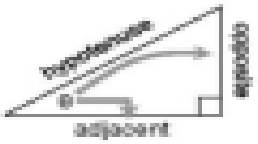
Essentials of Trigonometry 1

Usually in a geometry problem involving triangles, we have two pieces of information and we want a third piece. If we have two angles and want the third, we can use the fact that all three angles of a triangle add up to 180°. If we know two sides in a right triangle, and we want the third, we can use the Pythagorean theorem. If two of those three things are sides and one is an angle, then we must use trigonometry.

If I tell you that a right angle triangle has a 40° angle, that doesn't tell you what size the triangle is, but it does tell you the shape. All right triangles that have a 40° angle have the same shape, and their sides are proportional to each other. The triangle might be half as big as another, or twice as big, or ten times as big, but the ratio between the sides of the triangles will remain constant because the scaling factors will cancel out when we divide the lengths of the sides.



When taking the ratio of two of the sides, there are three different pairs of sides to choose from. To identify these pairs, we name the sides of the triangle. The longest side, which is always across from the right angle in the triangle is called the **hypotenuse**. For the other two sides, the name depends on which angle in the triangle we're looking at. When doing trigonometry with right triangles, we always do the calculations with *one of the two acute angles in mind*. If we're looking at the angle in the illustration above that's marked with a "θ" (theta), then the side at the bottom of the triangle is right next to that angle. In trigonometry, we say that it is adjacent to the angle, and it is called the **adjacent** side. The remaining side is across the triangle from angle θ. We say that it is opposite the angle, and so it is the **opposite** side. If we were working with the angle at the top of the triangle instead, the labels "adjacent" and "opposite" would switch places, but the hypotenuse wouldn't move. Which labels the two sides get in relation to the angle will tell you which tool to use to solve the problem. There are six possible ratios you can get from dividing two sides of a right triangle, and they all have names:

sine $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ **cosine** $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ **tangent** $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

cosecant $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$ **secant** $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$ **cotangent** $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

The first three functions, sine, cosine and tangent, all have buttons on your calculator while the other three don't. This is because for geometry, the last three are somewhat redundant — you can find them by calculating 1 ÷ sin, 1 ÷ cos and 1 ÷ tan. They are still necessary for calculus and other applications, so they should still be studied.

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