Calculus III: vectors worksheet

In questions 1 through 12, let

$$A = (1, 1), \qquad B = (2, -1), \qquad C = (0, 5),$$
$$D = (-1, 6), \qquad E = (0, 0), \qquad F = (\sqrt{3}, \pi).$$

1. Using a single coordinate plane $\mathbb{R}^2$ for each part (a), (b), (c), (d), sketch *and label* each of the following sets of points (vectors):

   (a) A, B, C, D, E, F.

   (b) $3A$, $A + C$, $B - D$, $3A + D$, $E + F$, $2B + C$.

   (c) $A + B$, $A + 2B$, $A + 3B$, $A + \frac{1}{2}B$, $A - B$, $A + \sqrt{2}\,B$.

   (d) The point midway between the points B and D.

2. Determine the six lengths $\|A\|, \ldots, \|F\|$.

3. The vector B displaces the point $(5, 7)$ to _____ and the point $(-9, -4)$ to _____.

4. What point is displaced by the vector B to the point $(1, 3)$? What point is displaced by B to $(0, 0)$?

5. What vector displaces the point $(4, -1)$ to the point $(3, 3)$? What vector displaces the point D to the point C?

6. The vector that displaces $(-3, 2)$ to $(6, 3)$ also displaces $(-8, -6)$ to _____ and $(6, 3)$ to _____.

7. Show that the point midway between B and D is $\frac{1}{2}B + \frac{1}{2}D$.

8. Find the point on the line from D to A that is one-quarter of the way from D to A; find the point that is *three*-quarters of the way.

9. **Triangle inequality** In Euclidean geometry, the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

   (a) Draw the triangle whose sides correspond to the vectors A, B, and $A + B$.

   (b) Compute $\|A + B\|$ to demonstrate $\|A + B\| < \|A\| + \|B\|$ (thus verifying the triangle inequality in this case, at least).

   (c) Find non-zero vectors X and Y for which $\|X + Y\| = \|X\| + \|Y\|$.

   (d) Does this contradict the triangle inequality? In fact, the "triangle" with sides X, Y, and $X + Y$ has a special (i.e., unusual) shape; what does it look like?

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