Solving Fractional Equations

Mathematicians don't like working with fractions any more than students do. When faced with a problem involving fractions, a great strategy for solving is to get rid of the fractions.

First, find the lowest common multiple for all the denominators of the fractions in a question. If some of the denominators are polynomials, it is useful to factor them when possible, and leave them factored until the end of the problem. Next, multiply the entire equation by the LCM. Cancel anything that can be cancelled, and you shouldn't have fractions anymore. At the end of the problem, check your solution against the original problem to make sure your solution is valid.

Example1: Solve $\frac{x^2}{x-1} = \frac{1}{x-1}$.

Solution: We already have a common denominator. Multiply both sides by $x - 1$:

$$\frac{x^2(x-1)}{x-1} = \frac{1(x-1)}{x-1}$$

$$x^2 = 1$$

$$x = \pm 1$$

It looks like we have two solutions, but we must check this against our original problem. A solution of $x = 1$ violates the original problem, since it gives us a denominator of 0. The solution, therefore, is $x = -1$.

Example 2: Solve $\frac{x+1}{x^2-x-6} = \frac{x-6}{x^2+2x-15}$.

Solution: We want to factor the polynomials that are in the denominators because it will be impossible to figure out the LCM if we don't.

$$\frac{x+1}{x^2-x-6} = \frac{x-6}{x^2+2x-15}$$

$$\frac{x+1}{(x+2)(x-3)} = \frac{x-6}{(x-3)(x+5)}$$

The LCM is $(x + 2)(x - 3)(x + 5)$. We don't need to reuse the common factor $(x - 3)$ in making the LCM.

$$\frac{(x+1)(x+2)(x-3)(x+5)}{(x+2)(x-3)} = \frac{(x-6)(x+2)(x-3)(x+5)}{(x-3)(x+5)}$$

$$(x+1)(x+5) = (x-6)(x+2)$$

$$x^2 + 6x + 5 = x^2 - 4x - 12$$

$$x^2 - x^2 + 6x + 4x = -12 - 5$$

$$10x = -17$$

$$x = -1.7$$

...which checks as the answer.

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