Set Notation & Interval Notation

Many algebra problems have a single solution. If we have $x + 3 = 8$, the only answer is 5, and we can simply write $x = 5$. Other problems have multiple solutions or a range of solutions. There are two main ways to report answers to a question like this: interval notation and set notation.

SET NOTATION

Set notation is useful especially when we have a small, finite number of solutions, rather than a range of solutions. Take the equation $x^2 = 9$. There are two answers: 3 and −3. The list of all possible solutions to a problem is called its **solution set** and we should write it as a set using roster notation: $\{-3, 3\}$. The curly brackets (brace brackets) indicate that the answer is a list and that −3 and 3 are the only two acceptable answers. A solution in roster notation can have more than two numbers in the brackets.

It's also possible to write the solution set to a problem by describing the solutions rather than by listing all of them. If we were asked, "What quantities of money can be withdrawn from a typical ATM?" and the ATM only dispenses $20 bills, then the answers are 20, 40, 60, 80, and so on. We could write $\{20, 40, 60, \ldots\}$ as a way of listing the answers, or we could use **set-builder notation** to say how to calculate the answers: $\{x \mid x = 20k, k \in \mathbb{N}\}$. We read this as: The solutions are x, where x is 20 times k, and k is a natural number. This is a very precise answer, and more precise than your teacher is likely to ask for.

INTERVAL NOTATION

Interval notation is used whenever the answers to a problem form one or more continuous ranges of the number line. This frequently happens in inequalities.

Take for example, $x^2 < 9$. After some thought, it should be obvious that any number between −3 and 3 (but not including either number) is a solution to the problem. We express this in interval notation by enclosing the numbers that are the endpoints of the solution in brackets. We use **round brackets** or parentheses when the interval does not include the endpoints, and **square brackets** when the interval does include the endpoints. Here, since the solution interval doesn't include those numbers on the end, we write: $(-3, 3)$. If the question were $x^2 \leq 9$, −3 and 3 would be valid solutions. We use square brackets to mark endpoints included in the solution: $[-3, 3]$. We can also use both bracket types in expressing a solution. For $4 < x \leq 7$, the interval runs from 4 to 7, and 4 is not a solution, but 7 is. We write: $(4, 7]$.

Sometimes there's no endpoint. For the question $x \geq 12$, there's a lowest possible solution, but no highest possible solution. We use the infinity symbol to show a lack of an endpoint, and we must always use a round bracket with it; infinity isn't a number, so it can't be a solution. We can't include it as part of a solution set. We write $[12, \infty)$. We use $-\infty$ for solutions with no lowest endpoint: $x \leq 12$ is expressed as $(-\infty, 12]$.

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