Properties of Real Numbers

To understand how algebra can be used to solve problems, we need to understand more about the sets of numbers we use, and the properties of the numbers in each one.

SETS OF NUMBERS

It helps to think of the historical context of different number systems. The way people used numbers has changed over time. It's only in the past century or so that we've begun to think of numbers without describing a quantity of things at the same time. "2 + 3 =" would not have been considered a practical problem; before doing the problem, most people would have asked, "Two what? Two apples? Two dollars?" They wouldn't think of numbers as an abstraction, only as a quantity of something they could hold or see.

Because of this, the oldest number system is the **natural numbers** or **counting numbers**. They were used to count things: three sheep, ten men, five years. The natural numbers represent quantities of things:

$$\{1, 2, 3, 4, 5, \ldots\}$$

In algebra, sets of numbers are represented by fancy "double-struck" letters. The natural numbers are represented by N, printed like this: $\mathbb{N}$, or hand-drawn like this: $\mathbb{N}$.

After the Dark Ages, a lot of lost mathematical and scientific information was re-introduced to Europe by the Moors. One mathematical concept they brought was the idea of nothing. If you had seven cattle and they all died, you had no cattle left, but before the 1200's this idea could not be expressed in numbers by most Europeans! When we include 0 with the natural numbers, we get **whole numbers**:

$$\mathbb{W} = \{\mathbf{0}, 1, 2, 3, 4, 5, \ldots\}$$

As banking and commerce increased, the next concept that changed the face of numbering systems was debt. If a merchant had promised to pay more money than he had, then it suddenly became very important to keep track of how much money the merchant owed. The amount that such a person owes is expressed as a **negative number**. When we put the negative numbers together with the whole numbers, we get **integers**. The integers are represented by the letter $\mathbb{Z}$:

$$\mathbb{Z} = \{\ldots \mathbf{-5, -4, -3, -2, -1,} 0, 1, 2, 3, 4, 5, \ldots\}$$

Why Z, rather than I? Z stands for the German name for integers: *die Zahlen*. The letter I is used for a set of numbers that you'll learn about later called imaginary numbers, which are numbers that are not real numbers.

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