

Factoring

The purpose of factoring is to make expressions easier to deal with. Essentially, we are writing large polynomials as a product of polynomials of a smaller degree. This is the opposite of multiplying polynomials. In general, terms that are multiplied together are easier to work with in fractions. Therefore, factoring can allow us to find terms to cancel in complicated fractions involving polynomials. Factoring is also good for finding where polynomials cross the x-axis (we will get to this later). It may be hard to believe now, but factoring (as complicated as it may seem) can really simplify many problems.

Common Factors

A common factor is an expression a where every term can be written as $a(\text{something})$. For example, $10x+5$ can be written as $5(2x)+5(1)$. Therefore, 5 is a common factor. Then, we can use the distributive property to write this as $5(2x+1)$, which is the factorization of $10x+5$. We want the greatest (biggest) common factor that goes in to every term. For example,

$$30x^2 + 6x + 12 = 6(5x^2) + 6(x) + 6(2) = 6(5x^2 + x + 2)$$

However, the greatest common factor is not always just a number. Sometimes, it has variables. In this case, it is important to recall the rule for exponents stating $a^x a^y = a^{x+y}$. So, if we have $3x^5y^3 + 6x^4y^4$, we see that 3 is a common factor. However, we have x and y in both terms. The rule is to take out the smallest power of each variable that appears in *any* term. So, we have x^5 in the first term, and x^4 in the second. Since x^4 has the smaller exponent, this is the biggest power of x that is common to both terms. Similarly, y^3 is the biggest power of y that is common to both terms. So, we have 3, x^4 and y^3 all as factors. So, the greatest common factor is the product of all of these terms, $3x^4y^3$. Putting it all together, we have $3x^4y^3(x^2) + 3x^4y^3(2y)$. Then, we can use the distributive property to get

$$3x^5y^3 + 6x^4y^4 = 3x^4y^3(x^2) + 3x^4y^3(2y) = 3x^4y^3(x^2 + 2y)$$

Factoring by Grouping

Sometimes, the entire expression has no greatest common factor that can be immediately factored out. However, it may be possible to find a common