Lie 2-groups & Lie 2-algebras

Thm. – Any Lie 2-group C determines a quadruple (G, H, t, α) where:

1) $G = C_0$ is the Lie group of objects
2) $H = \ker s \subseteq C_1$ is the Lie group of morphisms $h : 1 \to g$ where $1, g \in C_0$
3) $t : H \to G$ sends $h : 1 \to g$ to g.
4) $\alpha : G \to \mathrm{Aut}(H)$ is given by
$$\alpha(g)\, h = \mathrm{id}_g \cdot h \cdot \mathrm{id}_g^{-1}$$
mult. in C_1
not composition!

& this quadruple is a
Lie crossed module, meaning that also:
$$t\,(\alpha(g)\, h) = g \cdot t(h) \cdot g^{-1}$$
$$\alpha\,(t(h))\, h' = h\, h'\, h^{-1}$$

Conversely, any Lie crossed module determines a Lie 2-group. Similarly for Lie 2-algebras!