Factoring

I. **Greatest Common Factor**. Always check to see if you can factor out the greatest common factor (GCF). The greatest common factor is the largest factor that is shared by all the terms in the given expression. The GCF may include variables. Also, the GCF sometimes contains more than one term.

$10x^4 + 5x^3 - 20x^2$ ⟶ The GCF is $5x^2$.
$(x - y)(5x^2 + 6) - (x - y)(x^2 - 7)$ ⟶ The GCF is $(x - y)$.

After you determine the GCF, you may use the distributive property to rewrite the expression with the GCF factored out.

$8w^5 + 4w^3 + 24w$
$4w(2w^4) + 4w(w^2) + 4w(6)$ ⟶ Recognize 4w as the GCF.
$4w(2w^4 + w^2 + 6)$ ⟶ Use distributive property to write in factored form.

Now we will consider three types of polynomials: *binomial expressions* (two terms), *trinomial expressions* (three terms), *expressions with four terms*. The first step for all these cases will be to factor out the GCF.

II. Binomials. There are three special cases that fall under the two-term category.

A. **Difference of squares. $A^2 - B^2 = (A + B)(A - B)$** *This may be verified by multiplying out the right hand side.*

$25p^2 - 64$
$(5p)^2 - 8^2$ ⟶ This step may help you to *see* what the bases are.
$(5p + 8)(5p - 8)$ ⟶ Use the formula to rewrite in factored form.

B. **Difference of cubes. $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$** *Again, this may be verified by multiplying out the right hand side.*

$8p^3 - 64q^6$
$8(p^3 - 8q^6)$ ⟶ Factor out the GCF first.
$8(p^3 - (2q^2)^3)$ ⟶ Recognize the difference of cubes.
$8(p - 2q^2)(p^2 + 2pq^2 + 4q^4)$ ⟶ Write in factored form using the difference of cubes formula.

C. **Sum of cubes. $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$**

$64p^3 + q^3$
$(4p)^3 + q^3$ ⟶ Recognize the sum of cubes.
$(4p + q)(16p^2 - 4pq + q^2)$ ⟶ Write in factored form using the sum of cubes formula.

Note: There is no factorization for the sum of squares. For example, $9p^2 + 4q^2$ cannot be factored. It is prime.