Curve Sketching

A good graphing calculator can show you the shape of a graph, but it doesn't always give you all the useful information about a function, such as its critical points and asymptotes. Due to most graphing calculators' poor resolution, it can also be difficult to get detailed information about the shape of a graph. Curve sketching is a kind of analysis that determines useful information about a function and allows you to draw a remarkably accurate graph. The example below illustrates all the steps of curve sketching.

Example: Sketch the graph of $f(x) = \dfrac{x^2}{4x+1}$.

DOMAIN AND RANGE

If the function has a limited domain or range, then this should be the first thing you examine. Domain is more important for curve sketching than range. Rational functions, radical functions, logarithmic functions, and some trigonometric functions can have limited domains. Some trigonometric functions have restricted ranges.

The function's denominator cannot be 0. There are no other restrictions on x.

$$4x + 1 \neq 0$$
$$x \neq -\tfrac{1}{4}$$

The domain is $(-\infty, -\frac{1}{4}) \cup (-\frac{1}{4}, \infty)$.

INTERCEPTS

The x- and y-intercepts are the places where the function crosses the axes. These can be determined easily by evaluating the function at 0 (for x-intercepts) or solving the function for 0 (for y-intercepts). These points can be plotted directly onto the graph.

To find the y-intercept, we set x = 0:

$$\frac{[0]^2}{4[0]+1} = 0 \rightarrow (0, 0)$$

To find x-intercepts, we solve for $f(x) = 0$:

$$\frac{x^2}{4x+1} = 0$$
$$\therefore x^2 = 0$$
$$x = 0$$

There is only one x-intercept. (There can never be more than one y-intercept; do you know why?)

Authored by Darren Rigby