Division of Polynomials

We divide polynomials using a method similar to long division, so let's review that first.

$$\begin{array}{r} 35 \\ 7\overline{)246} \\ -21 \\ 36 \\ -35 \\ 1 \end{array}$$

In the long division process, you first find the largest multiple that you can for the first part of the dividend, subtract, bring down, and repeat this until you're done. The solution to the problem at the left might be written as $246 \div 7 = 35\frac{1}{7}$ or as $246 \div 7 = 35$ R 1.

Let's look at how polynomials are divided in a similar way.

Example 1: Divide $(x^3 - 2x^2 + 6x - 6) \div (x - 3)$.

Solution: First, we set up the division problem. Both the dividend and divisor should be written in decreasing powers of x (or whatever the variable for the question is). If there are any powers skipped, as in "$x^3 - x + 4$", then you must insert a placeholder term with a 0 coefficient where the gap is: "$x^3 + 0x^2 - x + 4$". Our problem looks like this:

$$x - 3\overline{)x^3 - 2x^2 + 6x - 6}$$

Just as in long division, we divide the first part of the dividend by the divisor. Here, our strategy is to get rid of the **leading term** (or first term) of the dividend on every step. We divide the leading term of the dividend (here, x^3) by the leading term of the divisor (x). We write the result over the term of the dividend with the same power. In our problem, we get x^2, so our division looks like this:

$$\begin{array}{r} x^2 \\ x - 3\overline{)x^3 - 2x^2 + 6x - 6} \end{array}$$

We then multiply this new term by the entire divisor, and write the result under the first part of the dividend, just like in long division:

$$\begin{array}{r} x^2 \\ x - 3\overline{)x^3 - 2x^2 + 6x - 6} \\ (-)\ \underline{x^3 - 3x^2} \end{array}$$

Then we subtract. The first term should always go away. If not, then something is wrong. Make sure to subtract the next column correctly: $-2x^2 - (-3x^2) = x^2$, not $-5x^2$. This is a common mistake! We start the next round of finding the quotient by bringing down the next term from the dividend and doing it all over again. In this example, the finished worked problem looks like this:

$$\begin{array}{r} x^2 + x + 9 \\ x - 3\overline{)x^3 - 2x^2 + 6x - 6} \\ (-)\ \underline{x^3 - 3x^2} \\ x^2 + 6x \\ (-)\ \underline{x^2 - 3x} \\ 9x - 6 \\ (-)\ \underline{9x - 27} \\ 21 \end{array}$$

Our quotient is $x^2 + x + 9$, and the remainder is 21. We can write this answer as:

$x^3 - 2x + 6x - 6 = (x - 3)(x^2 + x + 9) + 21$, or

$(x^3 - 2x + 6x - 6) \div (x - 3) = x^2 + x + 9 + \frac{21}{x-3}$.

We can check this by foiling out the quotient times the divisor, plus the remainder, which should equal the dividend.

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