Radicals

Radicals are sort of the opposites of exponents. The expression $\sqrt[3]{8}$ represents the number which must be multiplied 3 times to get an answer of 8, which in this case is 2. The parts of a radical expression have names:

$$\sqrt[5]{32}$$

For a **square root**—a radical with an index of 2—the two is usually not written. A radical with an index of 3 is a **cube root**, and radicals with an index of 4 or higher are fourth roots, fifth roots, and so on.

Radicals have laws that are similar to those for exponents:

Law:	Examples:
$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$	$25^{\frac{3}{2}} = \sqrt{25^3} = (\sqrt{25})^3 = 5^3 = 125$
$(\sqrt{x})^2 = x$ (when **x** is positive)	$(\sqrt{7})^2 = 7$
$\sqrt[n]{\mathbf{x}^n} = \mathbf{x}$ (when **n** is odd)	$\sqrt[3]{x^3} = x$; $\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = -2$
$\sqrt[n]{\mathbf{x}^n} = \lvert\mathbf{x}\rvert$ (when **n** is even)	$\sqrt{x^2} = \lvert x\rvert$; $\sqrt{x^4} = \lvert x^2\rvert = x^2$ **
**The absolute value sign is required for all variables in the answer that are raised to an odd power. The x in the answer to this second example is raised to an even power, therefore absolute value symbols are *not* required. (If the problem states that all variables are non-negative, then there is no need to worry about absolute value signs.)	
$\sqrt[n]{\mathbf{xy}} = \sqrt[n]{\mathbf{x}} \cdot \sqrt[n]{\mathbf{y}}$	$\sqrt{9x^4} = \sqrt{9} \cdot \sqrt{x^4} = 3 \cdot x^{\frac{4}{2}} = 3x^2$
$\sqrt[n]{\dfrac{\mathbf{x}}{\mathbf{y}}} = \dfrac{\sqrt[n]{\mathbf{x}}}{\sqrt[n]{\mathbf{y}}}$	$\sqrt[3]{\dfrac{27}{8}} = \dfrac{\sqrt[3]{27}}{\sqrt[3]{8}} = \dfrac{3}{2}$

ADDITION AND SUBTRACTION OF RADICALS

Only "like" radicals can be added or subtracted, i.e., radicals having the same index and the same radicand. When adding or subtracting, combine the coefficients and keep the "like" radical.

Example 1: Add: $6\sqrt[3]{2} + 10\sqrt[3]{2} - 5\sqrt[3]{2}$

Solution: $6\sqrt[3]{2} + 10\sqrt[3]{2} - 5\sqrt[3]{2} = (6 + 10 - 5)\sqrt[3]{2} = 11\sqrt[3]{2}$

Example 2: Simplify: $5 + 5\sqrt{3} + 3\sqrt{5}$

Solution: Since none of the terms have like radicals (the radicands are different), this expression cannot be simplified any more than it already is.

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