Factoring Secrets:

The ac-Method

This worksheet describes a method for factoring expressions of the type $ax^2 + bx + c$. This method is faster than the trial-and-error method.

Example 1: Factor $6x^2 - x - 2$.

Solution:

Step 1: Identify a, b, and c for the expression.

$a = 6$, $b = -1$, $c = -2$

Step 2: Multiply a by c (i.e. multiply the coefficient of the x^2 term by the constant term).

$ac = -12$

Step 3: Determine the possible pairs of factors that could yield the product a·c:

$-12 = -1 \times 12 = -2 \times 6 = -3 \times 4 = -4 \times 3 = -6 \times 2 = -12 \times 1$

Step 4: Decide which of the pairs of factors will add up to b.

$b = -1$, and $-4 + 3 = -1$.

Step 5: Replace the middle term by an equivalent expression using the two factors.

$6x^2 - x - 2 = 6x^2 + (-4x + 3x) - 2 = 6x^2 - 4x + 3x - 2$

Step 6: Factor by grouping.

$$\begin{aligned} 6x^2 - 4x + 3x - 2 &= (6x^2 - 4x) + (3x - 2) \\ &= 2x(3x - 2) + 1(3x - 2) \\ &= (2x + 1)(3x - 2) \end{aligned}$$

Example 2: Factor $6x^2 + 19x + 10$.

Solution:

Step 1: Identify a, b, and c for the expression.

$a = 6$, $b = 19$, $c = 10$

Step 2: Multiply a by c (i.e. multiply the coefficient of the x^2 term by the constant term).

$ac = 60$

Step 3: Determine the possible pairs of factors that could yield the product a·c:

$60 = 1 \times 60 = 2 \times 30 = 3 \times 10 = 4 \times 15 = 5 \times 12 = 6 \times 10$ and the negative versions of these.

Step 4: Decide which of the pairs of factors will add up to b.

$b = 19$, and $4 + 15 = 19$.

Step 5: Replace the middle term by an equivalent expression using the two factors.

$6x^2 + 19x + 10 = 6x^2 + 15x + 4x + 10$

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