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Expanding $\vec{E} + \vec{B}$ in powers of kr yields

$$\vec{E} \cong \hat{z} \frac{I_0}{2\pi a c \epsilon_0} \frac{\sin(\omega t)}{(ka/2)} \left[1 - \left(\frac{kr}{2}\right)^2 + \frac{1}{4}\left(\frac{kr}{2}\right)^4 + \dots \right]$$

$$\cong \hat{z} \frac{I_0}{\pi a c \epsilon_0} \frac{\sin(\omega t)}{ka} \left[1 - \left(\frac{kr}{2}\right)^2 + \frac{1}{2}\left(\frac{ka}{2}\right)^2 + \dots \right]$$

and

$$\vec{B} \cong \hat{\phi} \frac{I_0}{2\pi a c^2 \epsilon_0} \sin \omega t \frac{kr}{ka} \left[1 - \frac{1}{2}\left(\frac{kr}{2}\right)^2 + \dots \right]$$

$$\cong \hat{\phi} \frac{I_0}{2\pi a c^2 \epsilon_0} \sin \omega t \frac{r}{a} \left[1 - \frac{1}{2}\left(\frac{kr}{2}\right)^2 + \frac{1}{2}\left(\frac{ka}{2}\right)^2 \right]$$

b)

$$W_e = \frac{1}{2} \int \vec{E} \cdot \vec{D} dV = \frac{\epsilon_0}{2} \int_0^{2\pi} d\phi \int_0^d dz \int_0^a E^2(r) r dr$$

Using $\langle \sin^2 \omega t \rangle = \frac{1}{2}$ $c^2 = \frac{1}{\mu_0 \epsilon_0}$

$$W_e = \frac{d I_0^2}{2\pi \epsilon_0 c^2 a^2} \frac{1}{k^2 a^2} \int_0^a r dr = \frac{d I_0^2}{4\pi \epsilon_0 a^2 \omega^2}$$

(k²a² terms cancel)

$$W_m = \frac{1}{2} \int \vec{B} \cdot \vec{H} dV = \frac{1}{2\mu_0} \int_0^{2\pi} d\phi \int_0^d dz \int_0^a B^2(r) r dr$$

$$= \frac{\pi}{\mu_0} \frac{d I_0^2}{4\pi^2 a^3 c^4 \epsilon_0^2} \frac{1}{2} \int_0^a r^3 \left[1 - \left(\frac{kr}{2}\right)^2 + \left(\frac{ka}{2}\right)^2 \right] dr$$

$$= \frac{\mu_0}{4\pi} \frac{d I_0^2 a}{8} \left[1 + \frac{k^2 a^2}{12} \right]$$