

L3.1 The symbolic solution of $y' = -2xy$, $y(0) = 2$ is $y = 2e^{-x^2}$. Derive it and display a full answer check

Solution

$$y' + 2xy = 0$$

$$\frac{(WY)'}{W} = 0$$

$$(WY)' = 0$$

$$WY = c$$

$$y = c e^{-x^2}$$

$$2 = c e^0$$

$$\boxed{y = 2e^{-x^2}}$$

std form $y' + py = r$

$W =$ integrating factor $e^{\int p dx}$
 $W = e^{x^2}$

Quadrature

$$1/W = e^{-x^2}$$

$$\text{use } y(0) = 2$$

candidate solution

Answer check

$$\begin{aligned} \text{LHS} &= y' \\ &= (2e^{-x^2})' \\ &= 2(-2x)e^{-x^2} \\ &= (-2x)2e^{-x^2} \\ &= (-2x)y \\ &= \text{RHS} \end{aligned}$$

$$\text{use } (e^u)' = u'e^u$$

DE verified

$$\begin{aligned} \text{LHS} &= y(0) \\ &= 2e^{-x^2} \Big|_{x=0} \\ &= 2e^0 \\ &= 2 \\ &= \text{RHS} \end{aligned}$$

IC verified