

Mathematical Arguments, Theorems, & Proofs: Examples

Examples

1. Example 1: Let n be a positive integer. Prove that $n^2 + n$ is even.

Proof: Let n be a positive integer. We consider two cases: n is even and n is odd.

Case 1: Suppose n is even. Then $n = 2k$ for some integer k . Then $n^2 + n = (2k)^2 + 2k = 4k^2 + 2k = 2(2k^2 + k)$, which is even.

Case 2: Suppose n is odd. Then $n = 2k + 1$ for some integer k . Then $n^2 + n = (2k + 1)^2 + (2k + 1) = 4k^2 + 4k + 1 + 2k + 1 = 4k^2 + 6k + 2 = 2(2k^2 + 3k + 1)$, which is even.

In both cases, $n^2 + n$ is even. Therefore, $n^2 + n$ is even for all positive integers n .

Example 2: Let n be a positive integer. Prove that $n^2 + n + 1$ is odd.

Proof: Let n be a positive integer. We consider two cases: n is even and n is odd.

Case 1: Suppose n is even. Then $n = 2k$ for some integer k . Then $n^2 + n + 1 = (2k)^2 + 2k + 1 = 4k^2 + 2k + 1 = 2(2k^2 + k) + 1$, which is odd.

Case 2: Suppose n is odd. Then $n = 2k + 1$ for some integer k . Then $n^2 + n + 1 = (2k + 1)^2 + (2k + 1) + 1 = 4k^2 + 4k + 1 + 2k + 1 + 1 = 4k^2 + 6k + 3 = 2(2k^2 + 3k + 1) + 1$, which is odd.

In both cases, $n^2 + n + 1$ is odd. Therefore, $n^2 + n + 1$ is odd for all positive integers n .

Example 3: Let n be a positive integer. Prove that $n^2 + n + 2$ is even.

Proof: Let n be a positive integer. We consider two cases: n is even and n is odd.