

Calculus III: vectors worksheet

In questions 1 through 12, let

$$\begin{array}{lll} A = (1, 1), & B = (2, -1), & C = (0, 5), \\ D = (-1, 6), & E = (0, 0), & F = (\sqrt{3}, \pi). \end{array}$$

- Using a single coordinate plane \mathbb{R}^2 for each part (a), (b), (c), (d), sketch *and label* each of the following sets of points (vectors):
 - A, B, C, D, E, F .
 - $3A, A + C, B - D, 3A + D, E + F, 2B + C$.
 - $A + B, A + 2B, A + 3B, A + \frac{1}{2}B, A - B, A + \sqrt{2}B$.
 - The point midway between the points B and D .
- Determine the six lengths $\|A\|, \dots, \|F\|$.
- The vector B displaces the point $(5, 7)$ to _____ and the point $(-9, -4)$ to _____.
- What point is displaced by the vector B to the point $(1, 3)$? What point is displaced by B to $(0, 0)$?
- What vector displaces the point $(4, -1)$ to the point $(3, 3)$? What vector displaces the point D to the point C ?
- The vector that displaces $(-3, 2)$ to $(6, 3)$ also displaces $(-8, -6)$ to _____ and $(6, 3)$ to _____.
- Show that the point midway between B and D is $\frac{1}{2}B + \frac{1}{2}D$.
- Find the point on the line from D to A that is one-quarter of the way from D to A ; find the point that is *three-quarters* of the way.
- Triangle inequality** In Euclidean geometry, the sum of the lengths of any two sides of a triangle is greater than the length of the third side.
 - Draw the triangle whose sides correspond to the vectors A, B , and $A + B$.
 - Compute $\|A + B\|$ to demonstrate $\|A + B\| < \|A\| + \|B\|$ (thus verifying the triangle inequality in this case, at least).
 - Find non-zero vectors X and Y for which $\|X + Y\| = \|X\| + \|Y\|$.
 - Does this contradict the triangle inequality? In fact, the “triangle” with sides X, Y , and $X + Y$ has a special (i.e., unusual) shape; what does it look like?